*Field:* A ring R with identity 1 , where 1 0, is called a division ring (or skew field) if every nonzero element

a R has a multiplicative inverse, i.e., there exists

b R such that ab = ba = 1 . A commutative division ring is called a field. *D & F, p. 224.*

The *characteristic of a field F,* denoted ch(F), is defined to be the smallest positive integer p such that p · 1F = 0 if such a p exists and is defined to be 0 otherwise.

*D & F, p. 510.*

*field characteristic proposition 1\*:* The characteristic of a field F, ch(F), is either 0 or a prime p. If ch(F) = p then for any a F,

p · = + + · · · + = 0.

p times *D & F, p. 510.*

The *prime subfield* of a field F is the subfield of F generated by the multiplicative identity 1F of F. It is (isomorphic to) either (if ch(F) = 0) or Fp (if

ch(F) = p). *D & F, p. 511.*

field

field characteristic

field characteristic proposition 1\*

prime subfield

If K is a field containing the sub field F, then K is said to be an *extension field* (or simply an *extension*) of F, denoted K / F or by the diagram

K

|

|

F

In particular, every field F is an extension of its prime sub field. The field F is sometimes called the *base field* of the extension. *D & F, p. 511.*

The *degree* (or *relative degree* or *index*) of a field extension K/F, denoted [K : F], is the dimension of K as a vector space over F (i.e., [K : F] = dimF K). The

extension is said to be *finite* if [K : F] is finite and is said to be *infinite* otherwise. *D & F, p. 512.*

*field isomorphism proposition 2\*:* Let : F F' be a homomorphism of fields. Then is either identically 0 or is injective, so that the image of is either 0 or isomorphic to F. *D & F, p. 512.*

*isomorphic field theorem 3\*:* Let F be a field and let

p(x) F[x] be an irreducible polynomial. Then

there exists a field K containing an isomorphic copy of F in which p(x) has a root. Identifying F with this isomorphic copy shows that there exists an extension of F in which p(x) has a root. *D & F, p. 512.*