Field: A ring R with identity 1 , where $1 \neq 0$, is called a division ring (or skew field) if every nonzero element $a \in R$ has a multiplicative inverse, i.e., there exists $b \in R$ such that $ab = ba = 1$. A commutative division ring is called a field. *D & F, p. 224.*

field characteristic proposition 1:* The characteristic of a field F, ch(F), is either 0 or a prime p. If ch(F) = p then for any $a \in F$,

$$p \cdot \alpha = \underbrace{\alpha + \alpha + \cdots + \alpha}_{\text{p times}} = 0.$$

D & F, p. 510.

The *characteristic of a field F,* denoted ch(F), is defined to be the smallest positive integer p such that $p \cdot 1_F = 0$ if such a p exists and is defined to be 0 otherwise.
D & F, p. 510.

The *prime subfield* of a field F is the subfield of F generated by the multiplicative identity 1_F of F. It is (isomorphic to) either $\mathbb{Q}$ (if ch(F) = 0) or F_p (if ch(F) = p). *D & F, p. 511.*

field

field characteristic proposition 1*

field characteristic

prime subfield

If K is a field containing the sub field F, then K is said to be an *extension field* (or simply an *extension*) of F, denoted K / F or by the diagram

```
K
|
|
F
```

In particular, every field F is an extension of its prime sub field. The field F is sometimes called the *base field* of the extension. *D & F, p. 511.*

The *degree* (or *relative degree* or *index*) of a field extension K/F, denoted [K : F], is the dimension of K as a vector space over F (i.e., $[K : F] = \dim_F K$). The extension is said to be *finite* if [K : F] is finite and is said to be *infinite* otherwise. *D & F, p. 512.*

field isomorphism proposition 2:* Let $\varphi : F \to F'$ be a homomorphism of fields. Then φ is either identically 0 or is injective, so that the image of φ is either 0 or isomorphic to F. *D & F, p. 512.*

isomorphic field theorem 3:* Let F be a field and let $p(x) \in F[x]$ be an irreducible polynomial. Then there exists a field K containing an isomorphic copy of F in which p(x) has a root. Identifying F with this isomorphic copy shows that there exists an extension of F in which p(x) has a root. *D & F, p. 512.*