*Module:* Let R be a ring (not necessarily commut-

ative or with 1). A left R-module or a left module

over R is a set M together with

1. a binary operation + on M under which M is

an abelian group, and

1. an action of R on M (that is, a map R x M $\rightarrow $ M)

denoted by rm, for all r $\in $R and for all m $\in $ M

which satisfies

 (a) (r + s)m = rm + sm, for all r, s $\in $ R, m $\in $ M,

 (b) (rs)m = r(sm), for all r, s $\in $ R, m $\in $ M, and

 (c) r(m + n) = rm + rn, for all r $\in $ R, m, n $\in $ M.

 If the ring R has a 1 we impose the additional axiom:

 (d) 1m = m, for all m $\in $ M. *D & F,* p. 337.

*Submodule:* Let R be a ring and let M be an R-module.

An R-submodule of M is a subgroup N of M which is

closed under the action of ring elements, i.e., rn $\in $ N,

for all r $\in $ R, n $\in $ N.

 *D & F,* p. 337.

Modules over a field F and vector spaces over F are the

same.

 *D & F,* p. 337.

*Proposition 1:* Let R be a ring and let M be an R-module. A subset N of M is a submodule of M if and only if

(1) N$\ne ∅$, and

(2) x + ry $\in $ N for all r $\in $ R and for all x , y $\in $ N.

 *D & F,* p. 342.

module and vector space equivalence\*

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proposition 1

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module

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submodule

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